Ride Analysis of Three Wheeled Vehicle Using MATLAB/Simulink

M K Naidu[1], S Srinivasa Rao[2] and T Tejesh[3]
[1, 2]Asso.Profs., Mechanical Dept., M V G R College of Engineering
Email: mknaidu99@gmail.com and ssrinivasdme@gmail.com
[3]PG student, Mechanical Dept., M V G R College of Engineering
tejesh.tutaram@gmail.com

Abstract: **A spatial six degree freedom mathematical model of a three wheeled vehicle (TWV) used in Asian countries has been developed using multi body system approach. The model includes suspension and tyre compliance. The model consists of a single sprung mass (vehicle body) connected to three unsprung masses (three wheels) at each corner. The suspensions and tires are modeled as springs and linear damper elements. The model consists of six degrees of freedom because the body has three degrees freedom for bounce, pitch and roll motions and each unsprung mass have bounce motion. Vertical dynamic response of the TWV has been found when the vehicle is moving at 45 kmph on random road surface. Damping ratios and natural frequencies are obtained using Eigen value analysis. Ride analysis has been carried out in the frequency domain by performing the spectrum analysis using MATLAB/Simulink.**

Index terms: **three wheeled vehicle, random road profile, ride, frequency analysis, MATLAB/Simulink.**

I. Introduction

Three wheeled vehicles are extensively used for public transportation for small destinations in India and in many other countries of Asia. Three wheeled motor vehicles, typically used in India and most of the developing countries have their front steering with one wheel similar to those of motor cycles and two rear wheels are driving wheels with a differential and suspension, which are similar to those of automobiles.. The term ride is commonly used in reference to tactile and visual vibrations. The vehicle is a dynamic system, but only exhibits vibrations in response to excitation inputs.

The response properties determine the magnitude and direction of vibrations imposed on passenger's compartment. The understanding of ride involves the study of ride excitation sources and basic mechanics of vehicle vibration response. Ride comfort problem mainly arises due to surface irregularities.A mathematical model of a three-wheeled all-terrain vehicle(ATV)–rigid rider system without suspension with six degrees of freedom (DOFs) has been developed and simulation of ATV passing over three bump profiles, of rectangular, parabolic, and sinusoidal shapes, has been analyzed by Tan and Huston [1].The finite element stress analysis of three wheeler chassis [2] has been obtained under critical loads, simulating Indian road conditions by considering dead weight of vehicle, passengers, driver and ground excitations.Dynamic Vehicle response of a three wheeled motor vehicle in frequency domain has been studied using finite element modal. The response has been compared between the Indian and International roads and different modes which affect the passenger comfort [3].

A ride comfort simulation model based on the vibration of the two-mass system of vehicle body and wheels has been build and simulated for the vibration characteristics of the model by using simulation software MATLAB/simulink. The vehicle ride comfort is evaluated by comparison of the system parameters, such as natural frequency of vehicle body, damping ratio [4]. Vibration characteristics of vehicle without suspension and with front axle suspension were compared using 2DOF twin-shaft vehicle dynamic model and 3DOF twin-shaft vehicle dynamic model of front axle suspension vehicle using MATLAB/SIMULINK, with the white-noise for random excitation [5]. The dynamic response of the suspension of a road vehicle has been found using experimental setup fitted with dampers provided with strain gauges and simulated the behavior of the suspension of motor vehicles under the control of vibration using a model that more faithfully reproduces the actual behaviour. Simulation results in MATLAB /Simulink based on the mathematical model developed are compared with the experimental data and find a good concordance between experimental data and those provided by the mathematical model [6].

In this paper the mathematical model of TWV with suspension using multi body system approach is presented. Eigen value analysis has been carried out to find damped frequencies and ride characteristics have been studied in frequency domain by obtaining dynamic response under random road excitation using MATLAB/Simulink. Simulink is a software package for modeling, simulating, and analyzing dynamical systems. It supports linear and nonlinear systems, modeled in continuous time, sampled time, or a hybrid of the two. Systems can also be multirate, i.e., have different parts that are sampled or updated at different rates.For modeling, Simulink provides a graphical user interface (GUI) for building models as block diagrams, using click-and-drag mouse operations. This is more advantageous compared to other simulation packages that require formulating differential equations and difference equations in a language or program. Simulink includes a comprehensive block library of sinks,

© 2012 AMAE
DOI: 01.IJMMS.02.02.53

sources, linear and nonlinear components, and connectors. We can also customize and create our own blocks. Models are hierarchical, so we can build models using both top-down and bottom-up approaches. After defining a model, we can simulate it, using a choice of integration methods. Simulation results can be seen while simulation is running using scope and other display blocks.

In addition, we can change parameters and immediately see what happens, for "what if" exploration. The simulation results can be put in the MATLAB workspace for post processing and visualization. Model analysis tools include linearization and trimming tools, which can be accessed from the MATLAB command line, plus the many tools in MATLAB and its application toolboxes. And because MATLAB and Simulink are integrated, we can simulate, analyze, and revise our models in either environment at any point.

II. MODELING

The configuration of six degree of freedom is described in fig.1

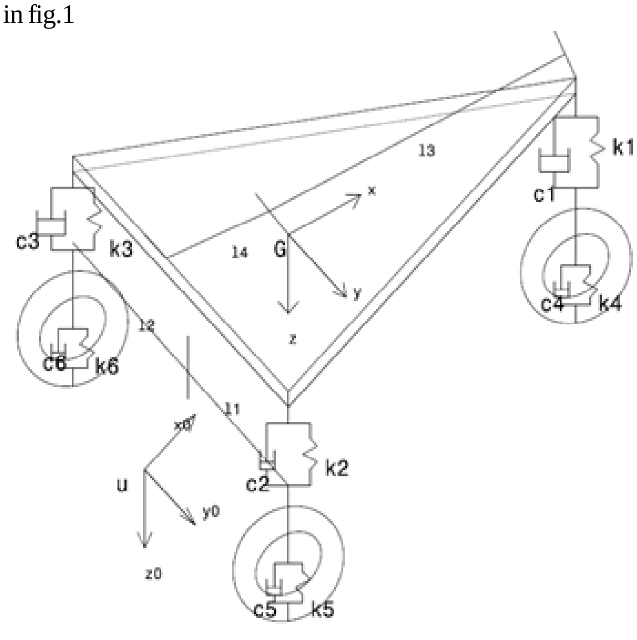

Fig 1. Discrete element model of the TWV

Vehicle attitude and trajectory through the course of maneuver are defined with respect to a right-hand orthogonal axis system, the inertial frame U-$X_0Y_0Z_0$ that is fixed to the earth. The origin of the moving reference frame G-*xyz* coincides with the centre of mass of the vehicle body and travels with the vehicle. The body of vehicle has 3DOF i.e. rotation about x and y axes (roll, pitch) and linear motion along z direction. Three wheels have linear motion in z direction (bounce). Three independent Euler angles are used to describe the orientation of the body-centered frame G-*xyz* in relation to the iner inertial frame. The transformation matrix is developed on the basis of these rotations. The Euler angles involve three successive rotations about three axes that are not orthogonal in general. The concept of yaw, pitch, and roll angles is used while selecting axes for rotations. The Newton's second law which states that sum of external forces acting on a body in a given direction is equal to the product of its mass and acceleration in that direction has been used for analysis of the system. Force balance equations have been derived for both translational and rotational systems. Euler transformations are used to simplify the equations. The final six degrees of freedom equations that has been obtained are

The bounce equation of motion for sprung mass:

$$m_s\ddot{z}+[c_1+c_2+c_3]\dot{z}+[c_3l_2-c_2l_1]\dot{r}+[c_2l_4+c_3l_4-c_1l_3]\dot{p}$$
$$-c_2\dot{w}_{rr}-c_3\dot{w}_{rl}-c_1w_f+[k_1+k_2+k_3]z+[k_3l_2-k_2l_1]r$$
$$+[k_2l_1+k_3l_4-k_1l_3]p-k_2w_{rr}-k_3w_{rl}-k_1w_f=0 \quad ..(1)$$

The roll equation of motion for sprung mass:

$$[I_{px}+m_sb^2]\ddot{r}+[c_3l_2-c_2l_1]\dot{z}+[c_2l_1^2+c_3l_2^2]\dot{r}+c_2l_1\dot{w}_{rr}$$
$$-c_3l_2\dot{w}_{rl}+[c_3l_2l_4-c_2l_1l_4]\dot{p}+[k_3l_2-k_2l_1]z+[k_2l_1^2+k_3l_2^2]r$$
$$+[k_3l_2l_4-k_2l_1l_4]p+k_2l_1w_{rr}-k_3l_2w_{rl}=0 \quad ..(2)$$

The pitch equation of motion for sprung mass:

$$I_{py}\ddot{p}+[c_2l_4+c_3l_4-c_1l_3]\dot{z}+[c_3l_2l_4-c_2l_1l_4]\dot{r}-c_2l_4\dot{w}_{rr}$$
$$[c_1l_3^2+c_2l_4^2+c_3l_4^2]\dot{p}-c_3l_4\dot{w}_{rl}+c_1l_3\dot{w}_f+[k_3l_2l_4-k_2l_1l_4]r$$
$$[k_3l_4+k_2l_4-k_1l_3]z+[k_1l_3^2+k_2l_4^2+k_3l_4^2]p-k_2l_4w_{rr}$$
$$-k_3l_4w_{rl}+k_1l_3w_f=0 \quad ..(3)$$

The bounce equation of motion of rear right wheel:

$$m_{rr}\ddot{w}_{rr}-c_2\dot{z}+c_2l_1\dot{r}+c_2l_1\dot{r}-c_2l_4\dot{p}+[c_2+c_5]\dot{w}_{rr}$$
$$-k_2z+k_2l_4p+[k_2+k_5]w_{rr}=0 \quad ..(4)$$

The bounce equation of motion of rear left wheel:

$$m_{rl}\ddot{w}_{rl}-c_3\dot{z}+c_3l_1\dot{r}-c_3l_4\dot{p}+[c_3+c_6]\dot{w}_{rl}$$
$$-k_3z+k_3l_2r-k_3l_4p+[k_3+k_6]w_{rl}=0 \quad ..(5)$$

The bounce equation of front wheel:

$$m_f\ddot{w}_f-c_1\dot{z}+c_1l_3\dot{p}+[c_1+c_4]\dot{w}_f-k_1z+k_1l_3p$$
$$+[k_1+k_4]w_f=0 \quad ..(6)$$

In the equations above *z, r, p* are bounce, roll, pitch displacements of sprung mass respectively and w_{rl}, w_{rr}, w_f are bounce displacements of rear left, rear right, front unsprung masses respectively. The variables that are having single dot '.' in super script are velocity components and variables having double dot are acceleration components of respective variables. The remaining variables and their numerical values that have been taken for simulation latter on in this paper are in table1.

© 2012 AMAE
DOI: 01.IJMMS.02.02.53

AMAE

TABLE I. NUMEERICLE VALUES FOR SIMULATION

Parameter	Variable	Value	Units
Sprung mass	m_s	492.3	Kg
Front unsprung mass	m_f	8.5	Kg
Rear right unsprung mass	m_{rr}	9	Kg
Rear left unsprung mass	m_{rl}	9	Kg
Front damping coefficient	C_1	3500	N.s/m
Rear right damping coeff	C_2	2207.5	N.s/m
Rear left damping coeff	C_3	2207.5	N.s/m
Rear right spring coeff	K_2	50400	N/m
Rear left spring coeff	K_3	49400	N/m
Front spring coeff	K_1	32700	N/m
Front tire damping	C_4	557	N.s/m
Rear right tire damping	C_5	436	N.s/m
Rear left tire damping	C_6	436	N.s/m
Front tire stiffness	K_4	238260	N/m
Rear right tire stiffness	K_5	250490	N/m
Rear left tire stiffness	K_6	250490	N/m
Rolling moment of inertia	I_{px}	182.2	Kg.m²
Pitching moment of inertia	I_{py}	170	Kg.m²
Right wheel track width from central plane	l_1	0.575	m
Left wheel track width from central plane	l_2	0.575	m
Distance between front tire and C.G	l_3	1.496	m
Distance between rear tire and C.G	l_4	0.504	m
Distance between C.G and roll axis	b	0.396	m

III. RANDOM ROAD EXCITATION

Road roughness is described by the elevation profile along the wheel tracks over which vehicle passes. One of the most useful representations is the power spectral density (PSD). A plot of the amplitude versus spatial frequency is the PSD. The relationship between the power spectral density and spatial frequency can be approximated by

$$S_g(\Omega) = C_{sp}\Omega^{-N} \qquad ..(7)$$

Where $S_g(\Omega)$ is the power spectral density function, C_{sp} and N are constants. Ω is the spatial frequency [4].

For vehicle vibration analysis, it is more convenient to express the power spectral density of surface profiles in terms of the temporal frequency in Hz rather than in terms of the spatial frequency, since vehicle vibration is a function of time. The transformation of the power spectral density of the surface profile expressed in terms of the spatial frequency $S_g(\Omega)$ to that in terms of the temporal frequency $S_g(f)$ is through the speed of the vehicle v.

$$s_g(\Omega) = \frac{s_g(f)}{v} \qquad ..(8)$$

IV. SIMULATION

Simulation has been carried out in MATLAB/Simulink when the vehicle is moving at 45 Kmph on high way with gravel road surface (C_{sp}=4.9x10^{-6} and N=2.1). In majority cases dynamic systems (that are continuous in time) will be described by differential equations. Thus in simulink we describe the system with a block diagram and simulate the reaction of the system to an input signal.

Road profile has been described as a PSD function (frequency domain) and the simulink has more emphasis on dynamic systems (time domain). Hence the road has to be generated as a random signal in time domain. The longitudinal positions of the signal can be related to a vertical displacement of the random road profile at that particular point. Thus the simulation of vehicle response on simulink in involves generation of random road signal, constructing the model and running the simulation for desired time and finding the response in frequency domain.

Random road profile has been generated in time domain in MATLAB workspace using sinusoidal approximation method [8, 9] in which a single track can be approximated by a superposition of N sine waves.

$$d(t) = \sum_{n=1}^{N} A_n \sin(n\omega_0 t - \phi_n), \qquad ..(9)$$

Where the fundamental temporal frequency

$$\omega_0 \cong V\Delta\Omega, \Delta\Omega \cong \frac{2\Pi}{L} \qquad ..(10)$$

And

$$A_n = \sqrt{\Phi(\Omega_n)\frac{\Delta\Omega}{\Pi}}, n = 1, \ldots N. \qquad ..(11)$$

The signal that has been generated from equation (9) has been given as input signal to the model. The simulink model has been constructed using block libraries. Each mode has been modeled separately in different sub systems and they are connected using bus creators and bus selectors. The parent diagram simulink modal is as shown in fig 2. The random road disturbance has been given as input to the system at three wheels. The accelerations of different modes are obtained in time domain.

The responses obtained in time domain are exported to MATLAB workspace and they are transformed into frequency domain using fast Fourier transforms.

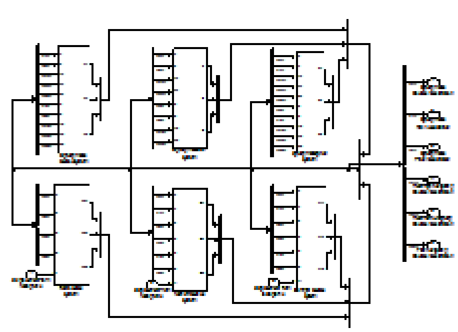

Fig 2. Parent diagram of simulink model

© 2012 AMAE
DOI: 01.IJMMS.02.02.53

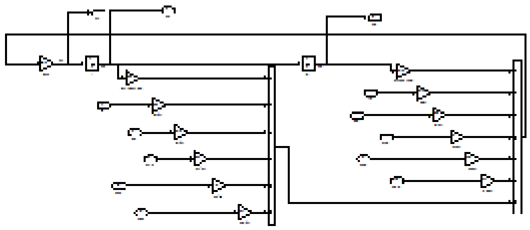

Fig 3. Simulink model for sprung mass bounce mode

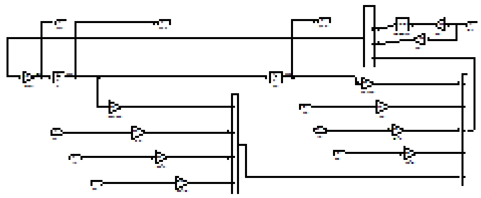

Fig 4. Simulink model for rear left unsprung mass system

RESULTS AND DISCUSSIONS

In the present work ride characteristics has been studied for the TWV in frequency domain when the vehicle is moving at 45 kmph on with gravel road profile. Natural frequencies have been found using Eigen value analysis. Out of twelve Eigen values that have been obtained from Eigen value analysis, two of them are found to be real and negative. . The remaining ten are complex occurring in conjugate pairs representing five oscillating modes. Out of five oscillatory modes of vibration, three of them are identified as roll, pitch and bounce modes for sprung mass system and remaining two are for unsprung mass systems. The damped frequencies and corresponding damping ratios for different modes that have been obtained from Eigen value analysis are in table 2.

TABLE II. DAMPED FREQUENCIES OF SIGNIFICANT MODES

S.NO	MODE DESCRIPTION	DAMPED FREQUENCY (HZ)	DAMPING RATIO
1	Sprung mass bounce	2.47	0.35
2	Sprung mass roll	1.73	0.24
3	Sprung mass pitch	5.94	0.64
4	Rear right unsprung mass bounce	17.21	0.804
5	Rear left unsprung mass bounce	17.21	0.804
6	Front unsprung mass bounce	16.02	0.829

The response of the vehicle i.e. power spectral densities of acceleration of heave, roll, pitch of sprung mass are as shown in fallowing figures.

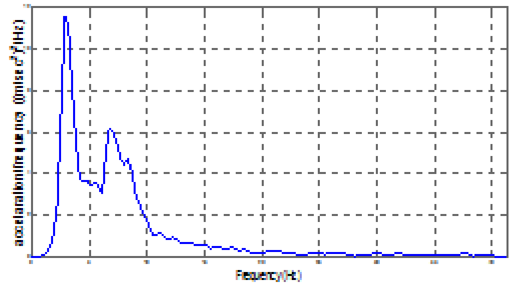

Fig 5. PSD of Acceleration of sprung mass bounce

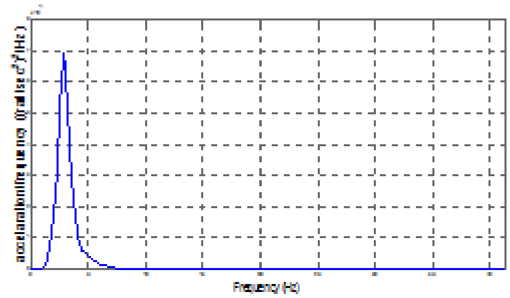

Fig 6. PSD of acceleration of sprung mass roll

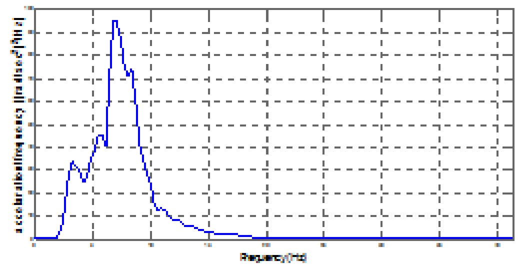

Fig 7. PSD of acceleration of sprung mass pitch

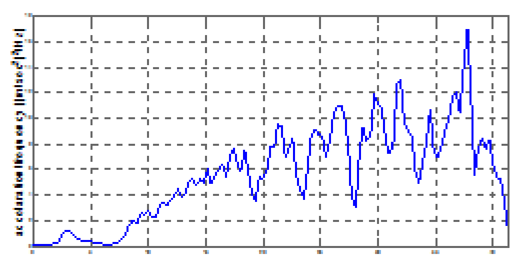

Fig 8. PSD of acceleration of rear right unsprung mass

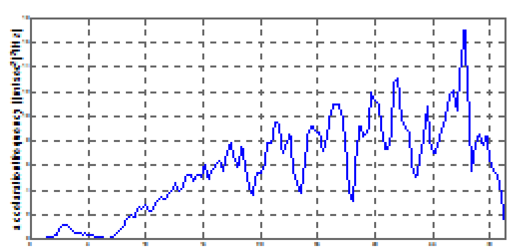

Fig 9. PSD of acceleration of rear left unsprung mass

© 2012 AMAE
DOI: 01.IJMMS.02.02.53

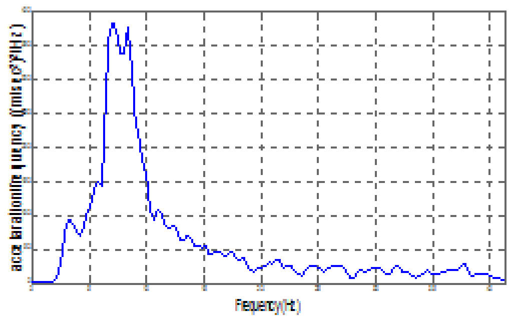

Fig 10. PSD of acceleration of front unsprung mass

From the results of sprung mass we can easily observe enhanced response at certain natural frequencies. Roll is having less impact on vertical vibrations compared to that of pitch. The acceleration of pitch has its maximum at 6 Hz and we can see increase in sprung mass bounce at the same frequency. Pitch mode is a more annoying when compared with other modes because its effect directly transferred to the passenger in the vehicle and they feel discomfort. PSD of acceleration of bounce of sprung mass has a maximum value at 2.8Hz. There a considerable increase in the acceleration in the range of 4Hz-8Hz which is more annoying because passengers in the vehicle feel discomfort because in this frequency range the resonance of the human stomach occurs. The unsprung mass natural frequencies are about 8 times to that of sprung mass bounce frequency. The responses of unsprung masses are changing in entire frequency range and wheels are less damped compared to the sprung mass. Wheel natural frequencies are high they have less impact on human comfort. Ride assessment using ISO tolerance limits are as shown in fig11.

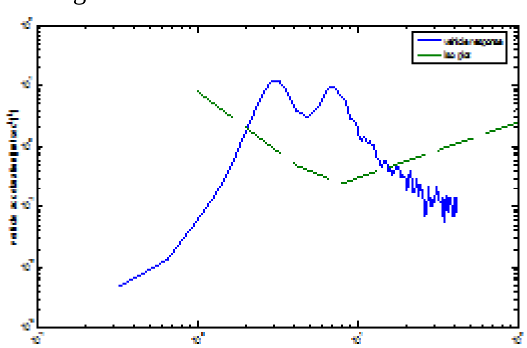

Fig 11. Comparison of vehicle vertical vibration with ISO tolerance limits

CONCLUSION

A spatial six degree freedom of TWV has been developed. Vertical dynamic response of vehicle has been obtained when the vehicle is moving at 45 kmph on random surface using MATLAB/Simulink. Ride analysis has been carried out in frequency domain. Damped frequencies and damping ratios has been found using Eigen value analysis.

REFERRENCES

[1] Tan,T.E., and Hutson j.c.,"three wheeled ATV-A No suspension system".SAE 1984 Trans.,paper no. 841058,93(4),1984,pp.4.806-4.817.

[2] Ramji. K, Vijay Saradhi. K, Sreenivasa Rao and Prabhakar B, "Dynamic Behavior of Three Wheeler Chassis, using Finite Element Method", Presented at National Conference on Future Trends in Mechanical Engineering, BIST, Chennai, Dec'2002.

[3] R Koona, MK Naidu "Dynamic Behavior of Three-wheeled Vehicle under Random Road Characteristics Using FEM", SEM annual conference, 2006.

[4] Yang, H.X.; Chen, F.M, "Ride comfort simulation based on vibrational characteristics of the two-mass system of vehicle body and wheels" International Conference on Computational Science and Engineering, 2009, CSE '09, pp-1044 – 1049.

[5] Baozhan Lu, Sihong Zhu, Ying Zhang, "Research on parameter matching of front axle suspension vehicle based on MAT LAB" IEEE International Conference on Automation and Logistics, 2007,pp 2441 – 2445.

[6] Marius-Constantin O.S, Popescu Nikos E. Mastorakis, "Testing and simulation of a motor vehicle suspension," International journal of systems applications, engineering & development, issue 2,vol3, 2009, pp. 74-83.

[7] J.Y Wong, Theory of ground vehicles, 3rd Edition, 1993, pp. 462-472.

[8] Feng Tyan and yu-Fen Hong "Generation of random road profile".CSME:B04-0001, 2006, PP. 1373-1377.

[9] G. Verros, S. Natsiavas, and C. Papadimitrioun, "Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation," Journal of Vibration and Control, vol. 11,pp. 581–606, 2005.

[10] Simulink, "Dynamic System Simulation for Use with Matlab," User's Guide, Math Works Inc., Natick, MA, 2004.

© 2012 AMAE
DOI: 01.IJMMS.02.02.53